OCR
Oxford Cambridge and RSA

Friday 17 June 2016 – Afternoon

AS GCE MATHEMATICS

4728/01 Mechanics 1

QUESTION PAPER

6394814653

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4728/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Book. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 A stone is released from rest on a bridge and falls vertically into a lake. The stone has velocity $14\,\mathrm{m\,s^{-1}}$ when it enters the lake.

(i) Calculate the distance the stone falls before it enters the lake, and the time after its release when it enters the lake. **[4]**

The lake is 15 m deep and the stone has velocity $20\,\mathrm{m\,s^{-1}}$ immediately before it reaches the bed of the lake.

(ii) Given that there is no sudden change in the velocity of the stone when it enters the lake, find the acceleration of the stone while it is falling through the lake. **[3]**

2 A particle P is projected down a line of greatest slope on a smooth inclined plane. P has velocity $5\,\mathrm{m\,s^{-1}}$ at the instant when it has been in motion for 1.6 s and travelled a distance of 6.4 m. Calculate

(i) the initial speed and the acceleration of P, **[5]**

(ii) the inclination of the plane to the vertical. **[3]**

3 Two forces each of magnitude 4 N have a resultant of magnitude 6 N.

(i) Calculate the angle between the two 4 N forces. **[4]**

The two given forces of magnitude 4 N act on a particle of mass m kg which remains at rest on a smooth horizontal surface. The surface exerts a force of magnitude 3 N on the particle.

(ii) Find m, and give the acute angle between the surface and one of the 4 N forces. **[3]**

4

Four particles A, B, C and D are on the same straight line on a smooth horizontal table. A has speed $6\,\mathrm{m\,s^{-1}}$ and is moving towards B. The speed of B is $2\,\mathrm{m\,s^{-1}}$ and B is moving towards A. The particle C is moving with speed $5\,\mathrm{m\,s^{-1}}$ away from B and towards D, which is stationary (see diagram). The first collision is between A and B which have masses 0.8 kg and 0.2 kg respectively.

(i) After the particles collide A has speed $4\,\mathrm{m\,s^{-1}}$ in its original direction of motion. Calculate the speed of B after the collision. **[4]**

The second collision is between C and D which have masses 0.3 kg and 0.1 kg respectively.

(ii) The particles coalesce when they collide. Find the speed of the combined particle after this collision. **[3]**

The third collision is between B and the combined particle, after which no further collisions occur.

(iii) Calculate the greatest possible speed of the combined particle after the third collision. **[4]**

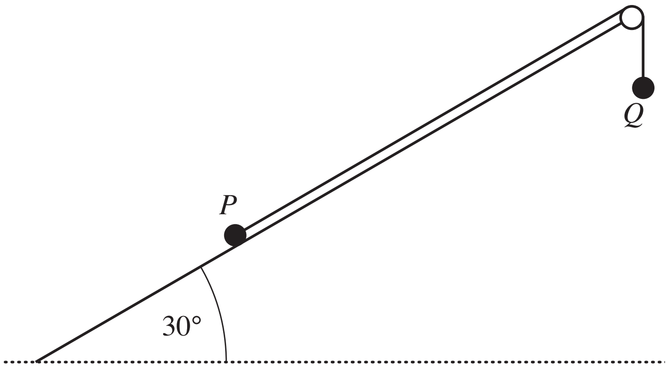
5 Three forces act on a particle. The first force has magnitude P N and acts horizontally due east. The second force has magnitude 5 N and acts horizontally due west. The third force has magnitude $2P$ N and acts vertically upwards. The resultant of these three forces has magnitude 25 N.

(i) Calculate P and the angle between the resultant and the vertical. **[7]**

The particle has mass 3 kg and rests on a rough horizontal table. The coefficient of friction between the particle and the table is 0.15.

(ii) Find the acceleration of the particle, and state the direction in which it moves. **[5]**

6



Two particles P and Q are attached to opposite ends of a light inextensible string which passes over a small smooth pulley at the top of a rough plane inclined at 30° to the horizontal. P has mass 0.2 kg and is held at rest on the plane. Q has mass 0.2 kg and hangs freely. The string is taut (see diagram). The coefficient of friction between P and the plane is 0.4. The particle P is released.

(i) State the tension in the string before P is released, and find the tension in the string after P is released. **[6]**

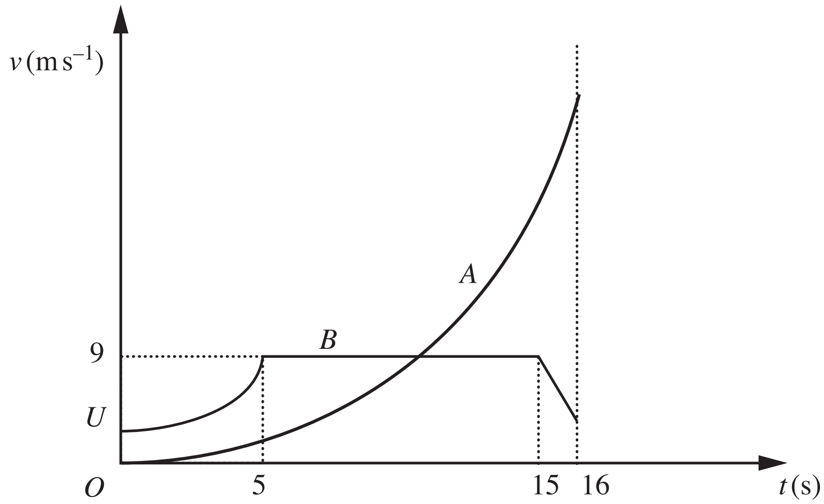
Q strikes the floor and remains at rest. P continues to move up the plane for a further distance of 0.8 m before it comes to rest. P does not reach the pulley.

(ii) Find the speed of the particles immediately before Q strikes the floor. **[5]**

(iii) Calculate the magnitude of the contact force exerted on P by the plane while P is in motion. **[3]**

Question 7 begins on page 4.

7



The diagram shows the (t, v) graphs for two particles A and B which move on the same straight line. The units of v and t are $\mathrm{m\,s^{-1}}$ and s respectively. Both particles are at the point S on the line when $t = 0$. The particle A is initially at rest, and moves with acceleration $0.18t\,\mathrm{m\,s^{-2}}$ until the two particles collide when $t = 16$. The initial velocity of B is $U\,\mathrm{m\,s^{-1}}$ and B has variable acceleration for the first five seconds of its motion. For the next ten seconds of its motion B has a constant velocity of $9\,\mathrm{m\,s^{-1}}$; finally B moves with constant deceleration for one second before it collides with A.

(i) Calculate the value of t at which the two particles have the same velocity. **[4]**

For $0 \leqslant t \leqslant 5$ the distance of B from S is $(Ut + 0.08t^3)\,\mathrm{m}$.

(ii) Calculate U and verify that when $t = 5$, B is $25\,\mathrm{m}$ from S. **[4]**

(iii) Calculate the velocity of B when $t = 16$. **[5]**

END OF QUESTION PAPER

Question			Expected Answer	Mark	Rationale/Additional Guidance
1	I		$14^2 = 2gh$ $h = 10$ m $14 = gt$ $t = 1.43$ s *OR* $14 = gt$ $t = 1.43$ s $h = 0\text{x}1.43 + 9.8\text{x}1.43^2 /2$ $h = 10(.0)$ m	**M1** **A1** **M1** **A1** **[4]** **M1** **A1** **M1** **A1**	$v^2=u^2 +/-2gs$ with $u=0$ -ve final answer A0 $v = u+gt$ with $u=0$ Accept 10/7 There are many alternatives, but following through of wrong answer is allowed only for method marks as the h and t values can be found independently.
	ii		 $20^2 = 14^2 + 2a15$ $a = 6.8 \text{ m s}^{-2}$	**M1** **A1** **A1** **[3]**	$v^2 = 14^2 +/- 2as$, $a \neq g$
2	i		 $6.4 = (u+5)/2 \text{ x } 1.6$ $u = 3 \text{ m s}^{-1}$ $5 = 3+1.6a$ $a = 1.25 \text{ m s}^{-2}$ *OR* $6.4=5\text{x}1.6-a1.6^2/2$ $a = 1.25 \text{ m s}^{-2}$ $5 = u + 1.25\text{x}1.6$ $u = 3 \text{ m s}^{-1}$	**M1** **A1** **A1** **M1** **A1** **[5]** **M1** **A1** **M1** **A1** **A1**	Uses $s=(u+v)t/2$ or a combination of two other formulae $5^2=u^2 + 2\text{x}6.4a$ M1 $5 = u + 1.6a$ M1 Accurate equation in one variable A1 $u = 3 \text{ m s}^{-1}$ A1 $a = 1.25 \text{ m s}^{-2}$ A1 Candidates may find a first (see below) $s = vt +/- at^2 /2$ Must be from $s = vt - at^2 /2$ **SC** Do not accept $a = 1.25$ from $6.4=5\text{x}1.6+a1.6^2/2$ but allow subsequent use of $a = 1.25$ in $5 = u + 1.25\text{x}1.6$
	ii		$1.25(m) = (m)g\text{CorS}\theta$ $1.25(m) = (m)g\cos\theta$ *OR* $1.25(m) = (m)g\sin\theta$ Angle with vertical = 82.7°	**M1** **A1**√ **A1** **[3]**	Resolves g or weight, $a \neq g$ ft cv(1.25) from (i) Must be angle with vertical

Question			Expected Answer	Mark	Rationale/Additional Guidance
3	i			**M1**	Resolve // Resultant
			$4\cos\theta + 4\cos\theta = 6$	**A1**	
			$\cos\theta = 6/8$	**M1**	
			Angle (= $2\theta = 2\cos^{=1}0.75$) = 82.8°	**A1**	
			OR	**[4]**	
			$6^2 = 4^2 + 4^2$ - 2x4x4cosα	**M1**	Cosine rule for triangle of forces
			α = 97.2°	**A1**	Cosine rule must give obtuse angle
			Angle = 180 – 97.2	**M1**	
			Angle = 82.8°	**A1**	Do not accept 82.8° from incorrect working
			OR		*OR*
			$6^2 = (4\sin\theta)^2 + (4+4\cos\theta)^2$	**M1**	$6^2 = (4\cos\theta)^2 + (4+4\sin\theta)^2$
			$36 = 16 + 32\cos\theta + 16$	**A1**	$36 = 16 + 32\sin\theta + 16$ hence $\theta = 7.2^\circ$
			$\cos\theta$ =4/32	**M1**	θ = 90-7.2
			$\theta = 82.8^\circ$	**A1**	$\theta = 82.8^\circ$.
	ii		*mg* = 6 + 3 *OR* *mg* = 4cos(Ans(i) /2) + 4cos((Ans(i) /2) + 3	**M1**	Must have signs correct
			m = 0.918	**A1**	
			Angle = 48.6°	**B1**√	Ft(90- cv(angle in (i)) /2)
				[3]	
4	i		0.8x6 - 0.2x2 (=4.4)	**B1**	Before momentum, signs different, no *g*
				M1	Uses momentum conservation, no *g*
			0.8x6 - 0.2x2 = 0.8x4 + 0.2*v* (= 4.4)	**A1**	
			$v = 6$ m s^{-1}	**A1**	*B*'s "after" velocity
				[4]	
	ii		After mass = 0.3+0.1	**B1**	
			0.3x5 (+0.1x0) = (0.3+0.1)*v*	**M1**	No *g*
			$v = 3.75$ m s^{-1}	**A1**	*CD* "after" velocity
				[3]	
	iii		Least final speed *B* = 4	**B1**	It cannot be less than the speed of *A*
			0.2x6+(0.3+0.1)x3.75 = 0.2x($v \geq 4$) + 0.4*V*	**M1**	Momentum, *B* and *CD* particles, essentially 4 non-zero terms with distinct velocities. Letters used at this stage should be checked against values used later.
			0.2x6+(0.3+0.1)x3.75 = 0.2x4+0.4*V*	**A1**√	ft cv (*v*(i) and *v*(ii))
			$V = 4.75$ m s^{-1}	**A1**	
				[4]	

Question			Expected Answer	Mark	Rationale/Additional Guidance
5	i		Perpendicular components of ($2P$) and +/-(5-P) $(P-5)^2 + (2P)^2 = 25^2$ $5P^2 - 10P - 600 = 0$ P=12 cosθ = (2x12)/25, tanθ = (12-5)/ 2x12 etc. Angle with vertical = 16.3°	**B1** **M1** **M1** **A1** **M1** **A1**√ **A1** **[7]**	 Uses appropriate Pythagoras Attempt to solve 3 term QE "=0" Targets any relevant angle appropriately ft cv(P) Must be angle with vertical
	ii		R = +/-(3x9.8 - 2x12) *OR* R = +/-(3x9.8 – 25cos(Ans(i))) R = 5.4 N (may be implied) 12 – 5 - 0.15x5.4 = 3a *OR* 25sin(cv(θ(i)) – 0.15x5.4 = 3a a = 2.06 m s^{-2} Direction East	**M1*** **A1**√ **D*M1** **A1** **B1** **[5]**	Bracketed terms must have opposite signs ft 29.4-2xcv(P(i)) *OR* 29.4 -25cos(cv(θ(i)) N2L, cv(12) cv(5.4) should be acceptable Allow bearing (0)90°
6	i		T(before) = 0.2g = 1.96 Fr = 0.4x0.2$g$cos30 (=0.67896..) 0.2a = 0.2g – T Either correct 0.2a =T - 0.2$g$sin30 - 0.4x0.2$g$cos30 Both correct 2T = 0.2g + 0.2$g$sin30 + 0.4x0.2$g$cos30 T = 1.81 N	**B1** **B1** **M1** **A1** **M1** **A1** **[6]**	Evaluation not needed Evaluation not needed, but accept 0.68 $a \neq g$ 0.2g - T=T - 0.2$g$sin30 - 0.4x0.2$g$cos30 is M1A1 0.4a=0.2g - 0.2$g$sin30 - 0.4x0.2$g$cos30 is M1A1 Finding expression (2)T from two simultaneous equations in a and T. a = 0.7526.. m s^{-2}, but is not required
	ii		THIS CANNOT BE SOLVED USING a(i) 0.2a = +/- (0.2$g$sin30 + 0.4x0.2$g$cos30) a = +/-(8.2948..) v^2 = 2x8.29(48..)x0.8 *OR* 0 = u^2 - 2x8.29(48..)x0.8 v = 3.64 m s^{-1} or u = 3.64 m s^{-1}	**M1*** **A1** **A1** **D*M1** **A1** **[5]**	N2L with Fr and Weight component of P Omitting g, M1*A0A0, D*M1A0 possible Equations must lead to positive values for u^2, v^2
	iii		 R^2 = (0.2$g$cos30)^2 + (0.4x0.2$g$cos30)^2 R=1.83 N	**M1** **A1** **A1** **[3]**	Applies Pythagoras to Friction and Normal Reaction Omitting g, M1A0A0 possible

Question			Expected Answer	Mark	Rationale/Additional Guidance
7	i		A: v=∫0.18t dt v=0.18/2 t^2 (+c) 9=0.09t^2 t = 10	**M1*** **A1** **D*M1** **A1** **[4]**	Integration indicated by change in coefficient and increase in power
	ii		B: v = d(Ut+0.08t^3) /dt v = U+0.24t^2 9=U+0.24x5^2 U = 3 SB(5)= 3x5+0.08x5^3 SB(5) = 25 m AG	**M1*** **D*M1** **A1** **A1** **[4]**	Differentiation indicated by change in coefficient and reduction in power There are instances of solutions in which SB(5) =25 is used to show that U=3, and then demonstrate that SB(5) = 25. Such work can gain no marks. u = 3 without any supporting work. M0A0.
	iii		A: x=∫0.09t^2dt x=0.09t^3 /3 x(16)=0.03x16^3 x=122.88 (may be implied by later work) 122.88=25+10x9+(9+v)(x1) /2 v=6.76 m s^{-1} *OR* 122.88-25-10x9 =9x1+/-ax1^2 /2 Deceleration = 2.24 m s^{-2} v = 9 – 2.24x1 v = 6.76 m s^{-1}	**M1*** **D*M1** **A1** **M1** **A1** **[5]** **M1** **A1**	Integration of v(A) Accept 123 $s = ut$ +/- at^2 /2
			Total	**72**	